P3 Managing a Deer Population

The goal of this project is to compare the effects of two different hunting policies on the deer population in a rural area. You are to decide which policy will keep the deer population from going extinct. First we need to make a few basic assumptions about how the deer population behaves if there is no hunting. It seems reasonable to assume that the growth rate is proportional to the size of the deer population, at least when the population is small. As the number of deer increases, competition for food reduces the growth of the herd. Thus, instead of an exponential model with constant unrestricted growth rate (= birth rate - death rate) of r, which would lead to:

$$x(n+1) = x(n) + r\,x(n) = (1+r)\,x(n)$$

we will multiply the growth rate by the term $\left(1-\frac{x(n)}{N}\right)$, where N is the so-called carrying capacity. As $x(n)$ gets closer to N, this factor becomes smaller and smaller, thus reducing the growth rate very much. However, when the population is small, then the above term is close to 1, and we almost have the exponential model. Using the paradigm **new = old + change**, the model for the deer population (without hunting) is

$$x(n+1) = x(n) + r\left(1-\frac{x(n)}{N}\right)x(n) = x(n) + rx(n) - r\frac{x(n)^2}{N} = (1+r)x(n) - r\frac{x(n)^2}{N} \quad (*).$$

a) Define the meaning your input and output variables and indicate their units.

<u>Hunting policy I: (Fixed numbers per year)</u>

b) Your first task is to evaluate a hunting policy that will reduce the herd by a fixed number of deer per year. Assume that you, as the forestry and fisheries manager, have a fixed allotment of hunting licenses to give out per year. Per hunting license, 5 deer can be shot. Let k = # of hunting license per year. Adapt the model (*) to incorporate the effect of hunting and state any additional assumptions you have made (if any).

c) Let's now look at the effect of different values of k, the number of hunting licenses. You have determined that the area inhabited by the deer can carry at most 10000 deer, and that the unrestricted growth rate is r = 0.8. You decide to sell 300 hunting licenses. Using the general formula

$$x = f(x),$$

where $f(x)$ is the right hand side of the iterative model equation, you can determine the so-called equilibrium values for this system. The above equation results when you assume that the new value equals the old: $x(n) = x(n+1) = f(x(n))$. Use the function **SolveIt** or analytical methods to determine these special values for the deer population.

d) Now you will check what effect the hunting policy in part c) has on the deer population in the long run for different initial population levels. Use **IteratedValueSeq** and **ListGraph** to find out what happens if the initial deer population is

i) 9000 ii) 7500 iii) 5000 iv) 3000 v) 2500 vi) 1000.

What do these graphs tell you about the applicability of the given hunting policy? Can you detect a difference between the two equilibrium values you found in part c)?

e) Your successful simulation of what happens in the case of 300 hunting permits makes you wonder what happens if you allow more hunting, i.e., give out more hunting permits. You now try what happens for 400 hunting permits. Use appropriate initial values (one above and one below each equilibrium value) for the deer population to see what happens. Describe the effects of the hunting policy with 400 hunting permits.

f) It can be shown that the critical value for the number of hunting licenses is $k = 400$. What happens if k is bigger than 400? (Using **SolveIt** will result in complex solutions (as indicated by the constant **I**) meaning that there is no equilibrium value.) Can you explain why any policy with less than 400 permits is called a sustainable policy?

<u>Hunting policy II: (Fixed percentage per year)</u>

g) You will now evaluate a second hunting policy. Rather than letting the general public do the hunting, the forest service decides to control the deer population with their own staff. They decide to hunt a fixed percentage **p** of the deer population. Adjust the model without hunting (given in b)) to reflect this new hunting policy. State any additional assumptions you may have made (if any).

h) Find the equilibrium value(s) for the deer population if

i) $p = 0.3$ ii) $p = 0.4$ iii) $p = 0.5$

i) Determine what happens to the deer populations for each of the three hunting policies in part h) by looking at initial values above and below the equilibrium, and checking out the long-term behavior as before.

j) Your supervisor wants you to tell him which of the two strategies to use - public hunting or forest service hunting. In particular, the deer population should stabilize in the long run at about 5000 deer. Which policy should be used? Give detailed reasons for your decision to convince your supervisor of the soundness of your recommendation.