P2 Total Horsepower (of Prime Movers) in the US

You have one source of data available for this project:

- Growth of total horsepower of all prime movers of the United States 1900 - 1985

Your goal in this project is to find a model for the total horsepower of all prime movers in the US.

1. Exponential Model (using the data for 1900 - 1955)

Overview: The first model you will consider is an exponential or Malthusian model, which we have discussed in class. The basic assumption for this model is that the increase in horsepower is proportional to the currently available horsepower (Idea: For every person/company using a prime mover, there will be a certain percentage of friends/neighbors/competitors that will also buy a machine/engine.)

a) Enter the data for the years 1900 - 1955 and graph it. (Associate 1900 with $n = 0$.)

b) Define the meaning of your input and output variables and indicate their units. (Make sure these units are in agreement with the way you entered the data in part a).)

c) Use the paradigm of **new = old + change** to set up the iterative model equation.

d) The iterative model equation derived in part c) contains a parameter, the growth factor, which also shows up in the general solution. In order to predict the total amount of horsepower, you need to estimate this growth rate. Use the data for the years 1900-1955 to come up with a value for the growth rate per time unit (as chosen for your input variable).

e) Using the growth rate derived in part d), state the general solution for the analytic model derived in part c).

f) Fit an exponential model to the data for the years 1900-1955, using the function **ExpoFitGraph**. From the function **ExpoFitFunc**, read off the growth factor per time unit. Compare this to the value you derived in part d). Are they similar? very different?

g) You have now derived two different models: One model was derived from assumptions (in part e)) and the other using a least squares fit (in part f)). Compare these two models using the function **FitComp**, and decide which one is the better model.

h) Using the best model found in part g), predict the total amount of horsepower for 1985. How does your prediction compare to the actual data?

g) As you have seen in part h), the model prediction is not very good. Can you explain the reasons? What are, in general, the limitations of an exponential model?

2. Logistic Model (using the data for 1900-1985)

Overview: As you have seen in part 1, there are some serious limitations of the exponential model. You will now adapt the model to (hopefully) derive a more realistic model. This revised model follows the ideas of Verhulst, who introduced (in the context of population models) a dampening factor. Instead of assuming that the change in population is proportional to the size of the current population (which would lead to unlimited growth), he assumed that there was an upper limit to how large the population could grow, the so-called carrying capacity L. Calling the difference between carrying capacity L and current population "unused growth potential", he postulated that

the change in population is jointly proportional to the current population and the unused growth potential (*)

a) Enter the full data set (1900 - 1985) and plot it.

b) Fit an exponential model to the larger data set, using **ExpoFitGraph** and**ExpoFitFunc**. Compare this exponential fit to the one you derived in part 1 f). (How well do these functions fit the data, how do the functional expressions from **ExpoFitFunc** differ?)

c) From the graph determine what type of function should be fitted to the data to get a better fitting model. Give reasons for your answer (using shape of the graph context, numerical tests if available).

d) You will now analytically derive a logistic model. Translate the verbal statement (*) into a mathematical expression using the fact that a quantity x is jointly proportional to quantities y and z, if there is a constant c such that $x = c \cdot x \cdot y$. Using the paradigm **new = old + change**, state the iterative model equation for the logistic model.

e) To use the logistic model derived in part d) for predictions, the value of the carrying capacity L needs to be determined. Use the data or its graph to estimate where the total amount of horsepower will level off.

f) Fit a logistic model to this larger set of data (1900 - 1985). Compare the logistic fit with the exponential fit of part 2b) using **FitComp**.

g) Using the function **LogisticFitFunc**, predict the total amount of horsepower in the years 1986 - 1991. From the Statistical Abstracts (1994 or 1995, category Energy) (available in the library), find the actual values for these years. Compare your model predictions to the actual data. How well does your model do? (Attach a copy of the data for 1986 - 1991 to your project.)

h) Who might be interested in such a model and why?

Project Data:

Commencing in 1850 periodic surveys have been made by the federal government to determine the total horsepower of all prime movers in the United States. According to the Department of Commerce: "Prime movers are mechanical engines and turbines which originally convert fuels or force into work and power." The total horsepower of all prime movers in the United States in 1985 is categorized in Table 1.

Table 1: Total horsepower of all prime movers in the United States by category, 1985.

Reference: Statistical Abstract of the United States 1987, U.S. Department of Commerce, Government Printing Office, Washington, D.C.

Category	[million HP]	Category	[million HP]
Automotive	30792	Farms	358
Factories	65	Power plants	912
Mines	47	Aircraft	268
Railroads	58		
Merchant ships	29	Total	32529

Table 2: Growth of total horsepower of all prime movers of the United States 1900-1985.

Reference: Historical Statistics of the United States, Colonial Times to 1970; Statistical Abstract of the United States 1987, Department of Commerce, Government Printing Office, Washington, D.C.

Year	t	[million HP]
1900	0	64
1910	10	139
1920	20	435
1930	30	1664
1940	40	2773
1950	50	4754
1955	55	7158
1960	60	11008
1965	65	15096
1970	70	20408
1975	75	25100
1980	80	28992
1985	85	32529