

## P1 Adoption of Improved Pasture Technology in Uruguay

You have one source of data available for this project:

- Table 2.2.3 Growth of the number of ranchers adopting improved pasture technology (Jarvis, American Journal of Agricultural Economics **63**, 495-502)

Your goal in this project is to find a model for the total number of ranchers that have adopted the improved technology (and more general, a model for how adoption of new technology spreads).

### 1. Exponential Model (using the data for 1961 - 1968)

**Overview:** The first model you will consider is an exponential or Malthusian model, which we have discussed in class. The basic assumption for this model is that the increase in ranchers adopting the new technology is proportional to the number of ranchers currently using the new technology. (Idea: Every rancher that uses the new technology influences a certain percentage of friends/neighbors to use it too.)

- a) Enter the data for the years 1961-1968 and graph it. (Associate 1961 with  $n = 0$ .)
- b) Define the meaning of your input and output variables and indicate their units. (Make sure these units are in agreement with the way you entered the data in part a.)
- c) Use the paradigm of **new = old + change** to set up the iterative model equation.
- d) The iterative model equation derived in part c) contains a parameter, the growth factor, which also shows up in the general solution. In order to predict the number of farmers using the new technology, you need to estimate this growth rate. Use the data for the years 1961-1968 to come up with a value for the growth rate per time unit (as chosen for your input variable).
- e) Using the growth rate derived in part d), state the general solution for your analytic model derived in part c).
- f) Fit an exponential model to the data for the years 1961-1968, using the function **ExpoFitGraph**. From the function **ExpoFitFunc**, read off the growth factor. Compare this to the value you derived in part d). Are they similar? very different?
- g) You have now derived two different models: One model was derived from assumptions (in part e)) and the other using a least squares fit (in part f)). Compare these two models using the function **FitComp**, and decide which one is the better model.

- h) Using the best model found in part g), predict the number of farmers using the new technology for the year 1976. How does your prediction compare to the actual data?
- i) As you have seen in part h), the model prediction is not very good. Can you explain the reasons for the discrepancy between model and data? What are, in general, the limitations of an exponential model?

## 2. Logistic Model (using the data for 1961 - 1976)

**Overview:** As you have seen in part 1, there are some serious limitations of the exponential model. You will now adapt the model to (hopefully) derive a more realistic model. This revised model, a logistic model, follows the ideas of Verhulst, who introduced (in the context of population models) a dampening factor for growth. Instead of assuming that the change in population is proportional to the size of the current population (which would lead to unlimited growth), he assumed that there was an upper limit to how large the population could grow, the so-called carrying capacity  $L$ . Calling the difference between carrying capacity  $L$  and current population the "unused growth potential", he postulated that

**the change in population is jointly proportional to the current population and the unused growth potential (\*)**

- a) Enter the full data set (1961-1976) and plot it.
- b) Fit an exponential model to the larger data set, using **ExpoFitGraph** and **ExpoFitFunc**. Compare this exponential fit to the one you derived in part 1 f). (How well do these functions fit the data, how do the functional expressions from **ExpoFitFunc** differ?)
- c) From the graph determine what type of function should be fitted to the data to get a better fitting model. Give reasons for your answer (using shape of the graph context, numerical tests if available).
- d) You will now derive analytically a logistic model. Translate the verbal statement (\*) into a mathematical expression using the fact that a quantity  $x$  is jointly proportional to quantities  $y$  and  $z$ , if there is a constant  $c$  such that  $x = c \cdot y \cdot z$ . Using the paradigm **new = old + change**, state the iterative model equation for the logistic model.

- e) To use the logistic model derived in part d) for predictions, the value of the carrying capacity  $L$  needs to be determined. Use the data or its graph to estimate where the number of farmers using the new technology will level off. Can you think of other ways of getting a ballpark estimate for this carrying capacity?
- f) Fit a logistic model to this larger set of data (1961-1976). Compare the logistic fit with the exponential fit of part 2b) using **FitComp**.
- g) Using the function **LogisticFitFunc**, predict the number of farmers using the new technology in the years 1980, 1985, 1990, and 1995. Do you think these numbers are realistic?
- h) Finally, describe why a model about the adoption of new technology is relevant and who might be interested in such a model.

### **Project Data:**

**Table 2.2.3:** Growth of the number of ranchers adopting improved pasture technology in Uruguay. From Jarvis (1981)

Year	t [years after 1961]	Newly adopting ranchers	Cumulative adopting ranchers
1961	0	141	141
1962	1	120	261
1963	2	136	397
1964	3	300	697
1965	4	247	944
1966	5	501	1445
1967	6	615	2060
1968	7	1187	3247
1969	8	2037	5284
1970	9	1815	7099
1971	10	2455	9554
1972	11	1911	11465
1973	12	2302	13767
1974	13	911	14678
1975	14	320	14998
1976	15	475	15473