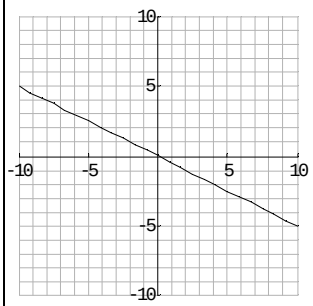
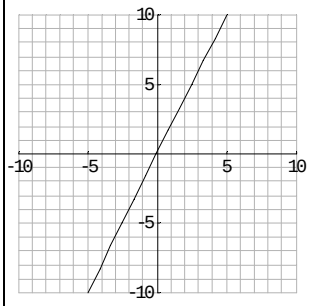
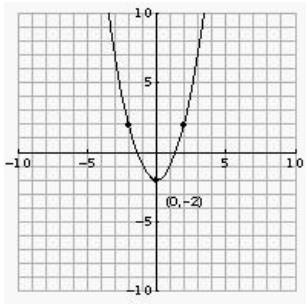
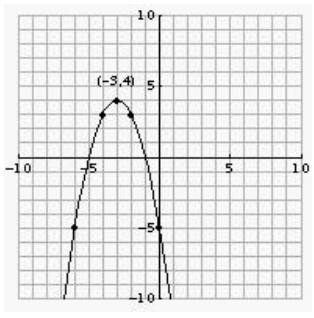
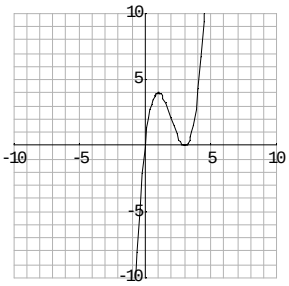
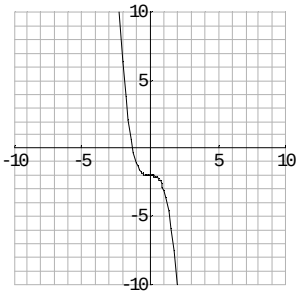
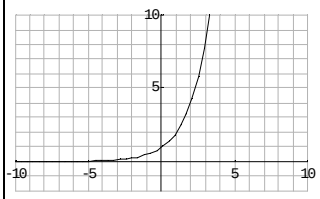
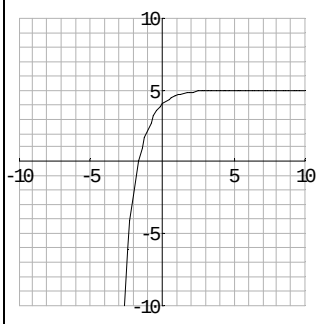
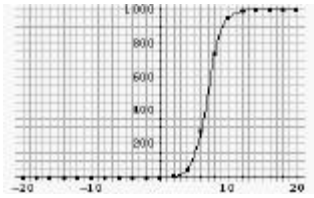
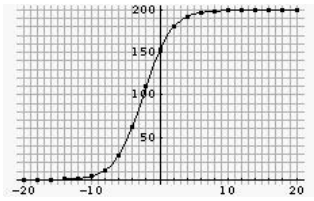
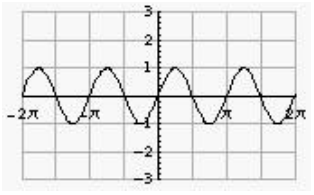
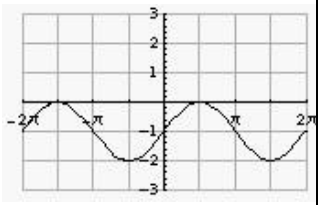
3.7 The Function Library

We will now summarize what we have learned about the specific function types described in Sections 3.1 through 3.6 in one convenient location. The first table contains the polynomial functions:

Function Type	**Linear**	**Quadratic**	**Cubic**
Formula	$f(x) = ax + b$	$f(x) = ax^2 + bx + c,$ $a \neq 0$	$f(x) = ax^3 + bx^2 + cx + d,$ $a \neq 0$
Shape of Graph	Straight line	Hill or bowl; no change of curvature	One change of curvature; values grow without bound in opposite directions (at the left end right end of the graph)
Examples of Graphs	 	 	 
Numerical Test	First unit differences are constant	Second unit differences are constant	None available

The next table shows the remaining three types of functions, namely exponential, logistic and sine functions.

Function Type	**Exponential**	**Logistic**	**Sine**
Formula	$f(x) = a \cdot b^x + c,$ $b > 0$	$f(x) = \frac{L}{1 + ae^{-bx}},$ $L > 0, a > 0,$ and $b > 0.$	$f(x) = a\sin(bx + c) + d,$ $a \neq 0$
Shape of Graph	No change of curvature; values grow without bound at one end of the graph and level off (horizontal asymptote) at the other end.	One change in curvature at level $\frac{L}{2}$; function grows between two horizontal lines (level 0 and L)	Periodic function; wavelike
Examples of Graphs	 	 	 
Numerical Test	Unit Ratios are constant for $f(x) = a \cdot b^x$	None available	None available

We are now ready for another lesson on *Mathematica*, namely how to work with palettes and packages, in particular the package **DataFit**. You will learn how to use some of the palette functions to graph the data and to perform the numerical tests for determining the function type (when available). Please open the *Mathematica* file L3.nb or refer to the section on *Mathematica* lessons.

Chapter Review

Key Terms

linear function	second unit difference	sine/cosine function
slope	cubic function	amplitude
first unit difference	polynomial of degree *n*	frequency
quadratic function	exponential function	period
parabola	constant unit ratios	vertical shift
vertex	logistic function	average (or middle value)

Short Answers

1. What is the interpretation of the slope *a* of a linear function in the problem context?
2. What is the difference between the two alternative forms of the quadratic function?
3. Describe the context in which a logistic function is likely.
4. Describe the context in which a sine/cosine function is likely.
5. Describe the steps used to determine which function type matches given data.

True - False Questions

T F 1. A linear function with equation $f(x) = a \cdot x + b$ intersects the horizontal axis at the point (*b*, 0).

T F 2. A quadratic function has constant unit ratios.

T F 3. An exponential function grows between two horizontal lines.

T F 4. There are no numerical tests for the logistic function.

T F 5. Cubic functions have two changes in curvature.

T F 6. A sine function is a polynomial.

T F 7. An exponential function is either always increasing or always decreasing.

T F 8. The amplitude of a sine function is computed by adding the largest and smallest output values, then dividing by 2.

T F 9. For small input values, the logistic function behaves like an exponential function.

T F 10. The period of a sine function is computed by subtracting the output values of consecutive points at which the function exhibits the same behavior (either increasing or decreasing).

Fill in the Blanks

1. The value of the slope *a* of a linear function determines the _____ of the straight line graph.
2. An exponential function of the form $f(x)$ = _____ has constant unit ratios.
3. The graph of a _____ function shows a repeated wavelike segment. The length of this repeated segment is called the _____.
4. For a quadratic function of the form $f(x) = ax^2 + bx + c$, the vertex has coordinates (_____,_____).
5. A logistic function has a change of curvature at output level _____ and levels off at the maximal capacity of _____.
6. The graph of a quadratic function is called a _____ and looks like a _____ or a _____.
7. A linear function has constant _____.
8. The following types of functions have no change of curvature: _____ and _____.
9. The _____ of a sine function measures how much the extreme high and low values deviate from the average value.
10. The value of the constant *d* in the equation of a sine function, $f(x) = a\sin(bx + c) + d$, gives the _____ or the _____.
11. A linear function is a polynomial of degree ____.
12. Identify the appropriate function type for each equation below.

    a. $f(x) = -2x + x^2$ _____

    b. $f(x) = \sin(3x - 2)$ _____

    c. $f(x) = \dfrac{300}{1 + 10e^{-0.9x}}$ _____

    d. $f(x) = 3x$ _____

    e. $f(x) = -30 \cdot 2^x$ _____

    f. $f(x) = -x^3 + 20$ _____