6.2 How to Manage Credit

Suppose you have a credit card with the *Students R Us* bank. When you started college, they gave you a credit card without requiring any credit history. You were eager to establish a credit history and jumped at the opportunity (and received the free mug and pens, too). Unfortunately, you have been using this credit card very heavily over the holidays and your current balance has soared to $5,649. Every time you open your monthly credit card statement, you see a list of purchases followed by an ever increasing amount of interest charged on the outstanding balance. You started this year with the firm commitment to eliminate your debt by paying a specified amount every month and by leaving the credit card at home, thus insuring you won't pile up any new charges. You are now wondering how long it will take to pay off the credit card. The fine print of your credit card statement indicates that the annual percentage rate charged for interest on the outstanding balance is 12.9%, resulting in a monthly interest rate of 1.075%.

In order to answer the question above, we need to derive a model for the credit card balance. First, we define the model variables and the initial condition. We are interested in the credit card balance (output variable) over time (input variable) and start out with a balance of $5,649. This leads to

$x(n)$ = balance (in $) n months from today

$x(0) = 5{,}649$

To use the paradigm **new** = **old** + **change**, we need to understand how the balance changes from month to month. The change in the credit card balance comes from three different sources: new charges, interest charged on outstanding balance, and the monthly payment. The first two increase the balance, the last one decreases the balance. However, since you decided not to use your credit card for purchases, the change in your credit card balance consists only of the interest charged and your monthly payment. Overall, we have:

new balance = **old** balance + **change**
= old balance + (interest - monthly payment)

If we denote the monthly payment by p, then we can compute the balance after one month as

$$\begin{aligned} x(1) &= x(0) + (0.01075 \cdot x(0) - p) \\ &= x(0) + 0.01075 \cdot x(0) - p \\ &= (1 + 0.01075) \cdot x(0) - p \end{aligned}$$

If we look at the balance after 2 months, we get

$$\begin{aligned} x(2) &= x(1) + (0.01075 \cdot x(1) - p) \\ &= x(1) + 0.01075 \cdot x(1) - p \\ &= (1 + 0.01075) \cdot x(1) - p \end{aligned}$$

In general, for the n^{th} month, the equation becomes

$$\begin{aligned} x(n) &= x(n-1) + (0.01075 \cdot x(n-1) - p) \\ &= x(n-1) + 0.01075 \cdot x(n-1) - p \\ &= (1 + 0.01075) \cdot x(n-1) - p \\ &= 1.01075 \cdot x(n-1) - p \end{aligned}$$

This is very similar to the model we had for the amount of drug in the patient's blood, except for the addition of a constant $(-p)$. Before we derive an explicit solution for this model, let's use the sequence tools of the TI-89 to get a feel for what is happening in the long run for different monthly payments.

Activity 6.2.1

Define a sequence describing the monthly balance, then create a table of values to read off the credit card balance after one year in the example described above if your monthly payment is

a) p = $25 per month. b) p = $50 per month c) p = $75 per month.

Describe what happens to your credit card balance by looking at the intermediate monthly balances. Will you be able to pay off your credit card with the monthly payments given in a), b), and c)? Can you find the “critical” payment amount for which the balance remains the same over time? Try to find this value by either using different payment amounts, or by applying the TI-89 function **solve** (which can be accessed by pressing [CATALOG] [S], then moving down to **solve**).

There are three qualitatively different behaviors:

- steadily increasing balance (for monthly payments of $25 or $50)
- constant balance
- steadily decreasing balance (for monthly payments of $75)

The second case is given a special name: if the output value, in our case the credit card balance, does not change over time, the system is said to be in *equilibrium*. In this case, the output value remains the same over time, i.e., $x(n-1)=x(n)$, no matter what the value of n happens to be. Therefore, we have the following equation:

$$\begin{aligned} x(n) &= a\cdot x(n-1)+b && \text{iterative model equation}\\ x(n) &= a\cdot x(n)+b && \text{replace } x(n-1) \text{ by } x(n) \end{aligned}$$

If we denote the equilibrium value by x instead of $x(n)$ (as it does not depend on n), we get

$$x = a\cdot x + b$$

This equation depends on three values, namely x, a, and b. If we know the values for any two of the three, we can solve the equation for the third quantity. In the question posed in Activity 6.2.1, we want to find the payment amount that keeps the balance the same. In this case, we know $a=1.01075$, the balance $x=5649$, and we want to find $b\ (=-p)$. The equation we need to solve is thus:

$$5649 = 1.01075\cdot 5649 + b$$

Solving it algebraically, we get

$$\begin{aligned} 5649-1.01075\cdot 5649 &= b\\ 5649\,(1-1.01075) &= b\\ -\underbrace{5649\cdot 0.01075}_{\text{interest paid per month}} &= b \end{aligned}$$

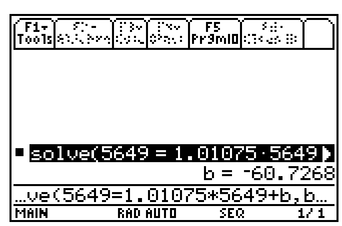
Thus, $p=-b=5649\cdot 0.01075$ = 60.7268. This makes a lot of sense--in order to keep a constant balance, we need to pay off exactly the amount of interest charged by the credit card company.

Alternatively, we could have used the TI-89 function **solve** to find b. Recall that the function **solve** has as its first argument the equation to be solved, and as its second argument the variable to be solved for (separated by a comma).

Press [HOME] to return to the Home screen. Press [CATALOG][S], then use the [2nd][▼] and [▼] keys to move to the function **solve**. Press [ENTER] and type

5649 [=] 1.01075 [×] 5649 [+] [alpha][B] [,] [alpha][B] [)]

followed by [ENTER].



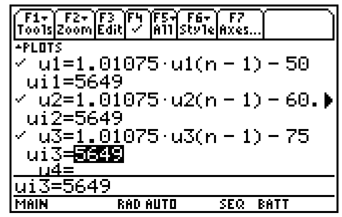
This result tells you that $b = -60.7268$, and thus the payment p (= -b) required to keep the balance constant is $60.73. (Why do we have to round up in this context?)

We will now summarize the three different behaviors mentioned before. Let's first look at a combined graph for the payment amounts $p = 50$, $p = 60.7268$ (the payment that ensures equilibrium), and $p = 75$.

1. Verify that the Graph mode is set to SEQUENCE→. Press [◆][Y=] and define the three sequences for the different payment amounts (you may already have done this in Activity 6.2.1). Recall that the model equation is given by

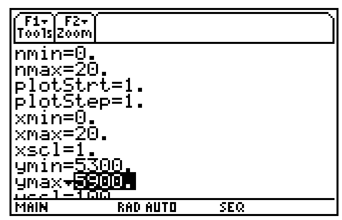
$$x(n) = 1.01075 \cdot x(n-1) - p$$

   and p = 50, p = 60.7268, and p = 75. In all three cases, the initial balance is 5649.

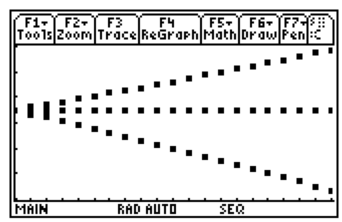


2. Press [◆][WINDOW] and set the parameter values as follows:

nmin = 0	nmax = 20	plotStrt = 1	plotStep = 1
xmin = 0	xmax = 20	xscl = 1	
ymin = 5300	ymax = 5900	yscl = 100	



3. Press [ENTER], then [◆][GRAPH] to see the three graphs together.



TIP:

When graphing several graphs together, you need to figure out which graph corresponds to which sequence. One way to do this is to graph each graph separately first (with the same window parameters as the joint graph). This can be quite tedious if more than two graphs are involved. In this case, you can use the trace feature. From the graph window, press [F3] to activate the trace. By default, it will be located on the first sequence. Pressing the ⊙ key will move the trace through the graphs in the same order in which the sequences are listed on the [Y=] screen.

From the graph, we can see the three behaviors:

- ever increasing balance if we pay $50 per month (which is less than the amount needed to balance the interest charged)
- constant balance, i.e., equilibrium, if we pay $60.7268 per month
- ever decreasing balance if we pay $75 per month (which is more than the amount needed to balance the interest charged).

We can conclude that if one pays less than the equilibrium payment of $60.7268, the credit card balance will increase every month and can never be paid off. On the other hand, paying any amount above the equilibrium payment will decrease the credit card balance every month and eventually the balance will be paid off.

We will now look at the credit card problem from a slightly different angle. In Activity 6.2.1, we started with a given initial balance and were asked what monthly payment is necessary to ensure the credit card can be paid off eventually. Now we will turn the question around: If you can make a monthly payment of a given amount, what is the largest balance that can be paid off with this amount?

Let's look at the following situation: You have been using your *Students R Us* credit card very responsibly in the past, mainly for emergencies. However, your mother's 50th birthday is coming up, and you would like to send her on a nice vacation. Unfortunately, you do not have enough cash and consider using your credit card for this purchase. You know that your part-time job will allow you to make monthly payments of $100. If the interest rate is 1.075% per month, what is the largest amount you can spend and still be able to eventually pay off your credit card balance?

Activity 6.2.2

Assume that your monthly interest rate is 1.075% and your payment amount is $100. Can you pay off your balance eventually if your current balance is

a) $10000? b) $9500? c) $9000

Use your answers to a) - c) to narrow down the "critical" balance, at which your payment of $100 exactly balances the interest charged. Use additional values for the balance or the TI-89 function **solve** to determine the exact critical balance. What happens if your credit card balance rises beyond that critical balance?