## 3.5 Logistic Functions

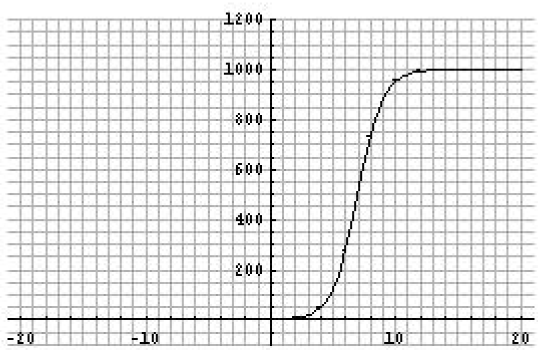
The general equation of a *logistic function* is given by

$$f(x) = \frac{a}{1+be^{c\cdot x}} + d \quad \text{with } a > 0 \text{ and } b > 0.$$

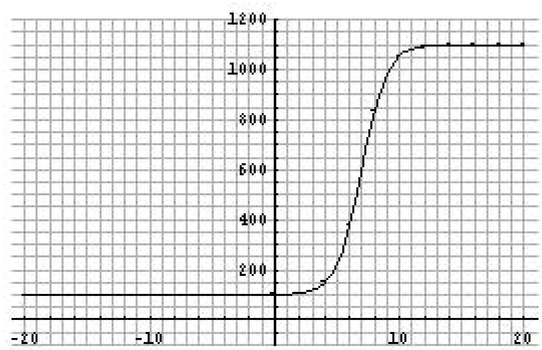
Graphs of logistic functions look like a stretched S-shape and are increasing or decreasing between two horizontal lines. There is one change in curvature. Data that follows an increasing logistic curve usually describes constrained growth or a cumulative quantity. For small values of the independent variable, the increasing logistic function behaves very much like an (increasing) exponential function. However, for large values of the independent variable, the two functions behave quite differently.

Below are a few examples of graphs of logistic functions. Note that even though the input values range from $-20$ to 20 in each case, the output values have quite different ranges.

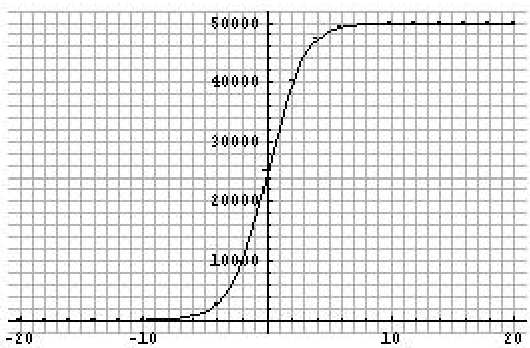
a) $f_1(x) = \frac{1000}{1+999e^{-0.9906x}}$



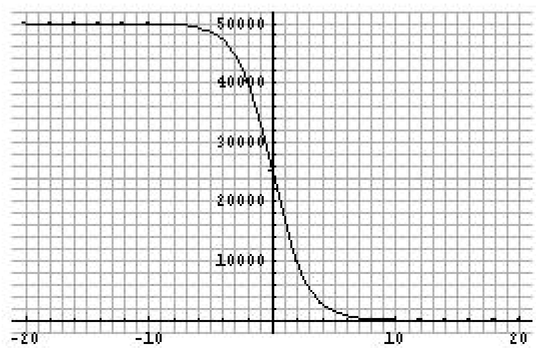

$f_2(x) = \frac{1000}{1+999e^{-0.9906x}} + 100$



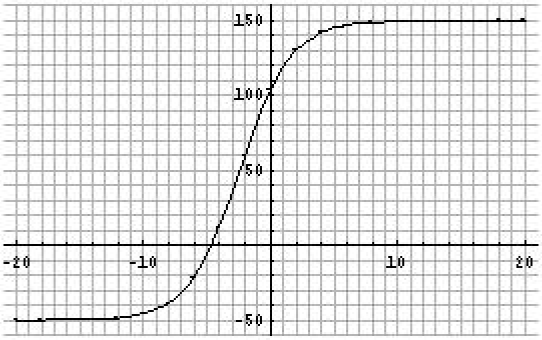

b) $n_1(t) = \frac{50000}{1+e^{-0.7033t}}$

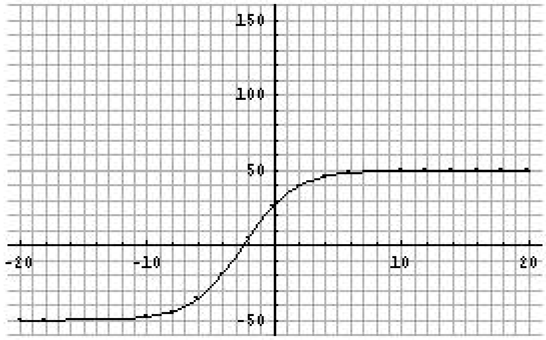



$n_2(t) = \frac{50000}{1+e^{0.7033t}}$



c) $h_1(u) = \dfrac{200}{1+0.3e^{-0.5u}} - 50$ $\qquad$ $h_2(u) = \dfrac{100}{1+0.3e^{-0.5u}} - 50$




**Activity 3.5.1**

For the function pairs given previously, identify the values of the parameters *a*, *b*, *c*, and *d*. For each pair of functions, only one parameter value changes; the corresponding change in the graph must come from this parameter. Use this fact to determine the influence of the values of *a*, *c*, and *d* on the graph. (In particular, look at increasing versus decreasing, the limiting values, and the output value of the point at which there is a change in curvature.)

a) $f_1(x) = \dfrac{1000}{1+999e^{-0.9906x}}$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

$f_2(x) = \dfrac{1000}{1+999e^{-0.9906x}} + 100$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

b) $n_1(t) = \dfrac{50000}{1+e^{-0.7033t}}$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

$n_2(t) = \dfrac{50000}{1+e^{0.7033t}}$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

c) $h_1(u) = \dfrac{200}{1+0.3e^{-0.5u}} - 50$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

$h_2(u) = \dfrac{100}{1+0.3e^{-0.5u}} - 50$ $\quad a =$ $\quad b =$ $\quad c =$ $\quad d =$

<u>Properties of Logistic Functions</u>

1) The functional expression of a logistic function is $f(x) = \dfrac{a}{1+be^{c \cdot x}} + d$ with $a > 0$ and $b > 0$.
2) The graph of a logistic function increases or decreases between two horizontal lines at levels *d* and *a* + *d*. The graph changes curvature once, at the point where the output value equals *a*/2 + *d*.
3) The function is increasing when *c* < 0, and decreasing if *c* > 0.

Checking for logistic functions is different from checking for the types of functions we've seen so far. Previously, we looked at two kinds of properties that existed: those that can be checked from the graph and those that can be checked numerically. Checking for logistic functions is purely based on the shape of the graph and knowledge of the context. No numerical tools are available.

**Activity 3.5.2**

The table below gives the number of European, North American, and South American countries issuing postage stamps from 1840 through 1880. Plot the data and determine whether it comes from a logistic function. Give a reason for your answer. From the data and the graph, estimate the value at which the function will level off. How could you verify your guess?

| Years | 1840 | 1845 | 1850 | 1855 | 1860 | 1865 | 1870 | 1875 | 1880 |
|---|---|---|---|---|---|---|---|---|---|
| Countries | 1 | 3 | 9 | 16 | 24 | 30 | 34 | 36 | 37 |