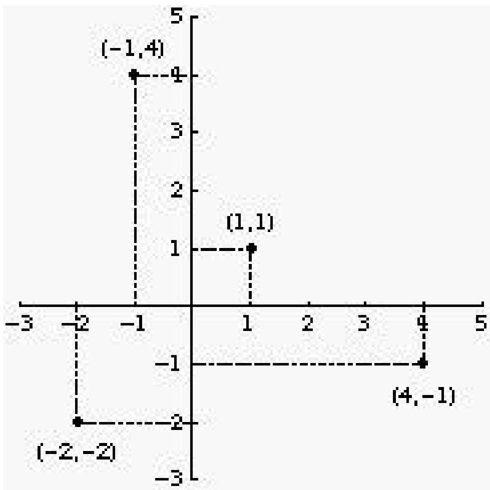
2.2 Graphs of Functions

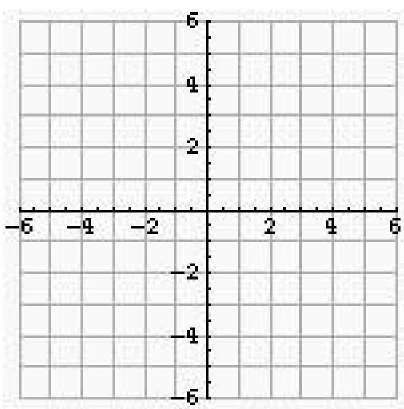
In addition to describing a function via a table or a function rule, we can visualize the function through a graph on the *Cartesian Coordinate Plane*. In the Cartesian Coordinate Plane, the horizontal and the vertical axes intersect in a right angle. Each axis has a scale with numbers increasing from left to right and bottom to top, respectively. The crossing of the two axes is where each scale has value 0 and is called the *origin*. Every point in the plane has two location parameters: one for the horizontal distance from the origin and one for the vertical distance from the origin. Customarily, the horizontal position is listed first, followed by the vertical position.

Below, several points have been plotted on the coordinate plane and are marked with the pair of location coordinates.



Activity 2.2.1

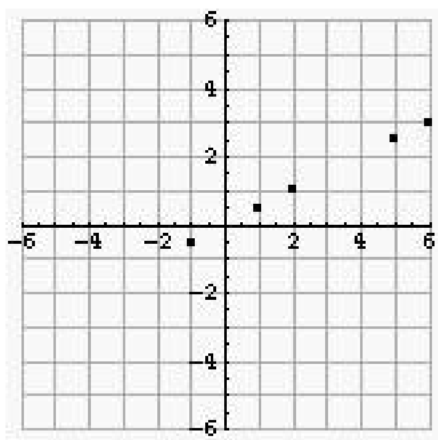
In the coordinate system below, plot (-1, 2), (5, 1), (3, -2), and (-2, -2.5).



To visualize a function, we create points from the table of values and plot them on the coordinate system. By convention, the input value is the first coordinate, and the output value is the second coordinate. Thus, the input value is marked off on the horizontal axis, and the output value on the vertical axis. For example, for the first function described in Section 2.1, the table would be translated into these points:

input x	output $f(x)$	points
-1	-0.5	(-1,-0.5)
1	0.5	(1,0.5)
2	1	(2, 1)
5	2.5	(5, 2.5)
6	3	(6,3)

Plotting the points onto the coordinate system gives the following graph:



Note that the rule $f(x) = \frac{1}{2}x$ works for many input values, not just for the ones given in the table. (What is the mathematical domain of this function?) If we were to plot points for additional input values, we would start to see a particular shape for the graph.

Activity 2.2.2

a) Make a table with additional input-output values for the function $f(x) = \frac{1}{2}x$.

input x	output $f(x)$	points

b) Plot these additional points on the coordinate system above and connect them.

c) What kind of shape does the graph of this function have?

You should have seen a straight line. In the next section, we will look at which function rules are associated with particular shapes such as straight lines, parabolas, periodic "wavy" curves and more.

Activity 2.2.3

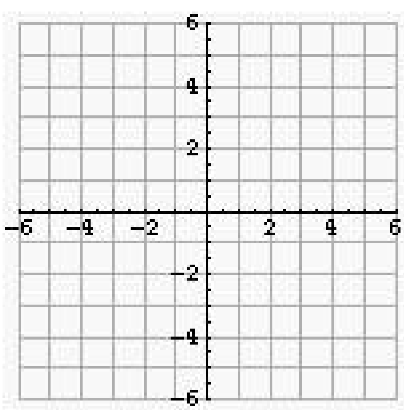
In the coordinate systems below, graph the following functions.

a) $f(x) = x^3$

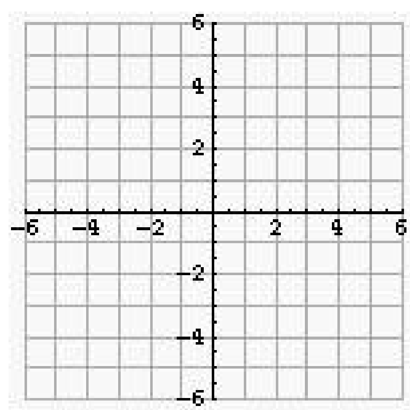
b) $f(x) = \begin{cases} x^2 & \text{if} \quad x \le 1 \\ -x & \text{if} \quad x > 1 \end{cases}$

You can use the values computed in Activity 2.1.3. Try to connect the points to see what curve will result. (You may have to compute additional input-output pairs to help you "see" the shape of the curve.)

a)



b)



One property of functions that can be verified very easily from a graph is its monotonicity, that is, whether the function is increasing or decreasing.

Definition:

A function is *increasing* if for any values x_1 and x_2 with $x_1 < x_2$ we have $f(x_1) \le f(x_2)$.
(Think: Larger input values result in larger output values).

A function is *decreasing* if for any values x_1 and x_2 with $x_1 < x_2$ we have $f(x_1) \ge f(x_2)$. (Think: Larger input values result in smaller output values).

Graphically, for an increasing function the output values increase from left to right; for a decreasing function, the output values decrease from left to right.

Activity 2.2.4

Determine whether the functions in Activity 2.2.3 are increasing, decreasing or neither. In the latter case, determine for which sets of input values the function is increasing or decreasing.

a)

b)

Chapter Review

Key Terms

function (rule)	mathematical domain	increasing function
functional expression	context domain	decreasing function
independent variable	Cartesian Coordinate Plane	
dependent variable	origin	

Short Answers

1. What is a function?
2. What is the difference between the dependent and independent variables?
3. How is the context domain different from the mathematical domain?

True - False Questions

T F 1. Given a table of input-output pairs, you can graph the data.
T F 2. A function rule can always be accurately determined from a set of data.
T F 3. A function rule can be determined from a table of values and vice versa.
T F 4. The independent variable is often denoted by x or t.
T F 5. The context domain of a problem is either the same as the mathematical domain or smaller.
T F 6. In an increasing function, larger input values create smaller output values.

Fill in the Blanks

1. Functions can be represented as a ________________, ________________, or ________________.
2. The set of numbers for which a function can be computed is the ________________ domain.
3. If a function is increasing, then ________________ input values result in ________________ output values.