# A3 Properties of Exponents

Exponents help us express repeating factors of a product in a compact way. Whenever a number or variable is multiplied by itself some number of times, we can write it in exponential form. The factor (=number or variable) is called the *base* and the number of times it is repeated is called the *exponent*. For example, $x^2$ has $x$ as its base and the exponent is 2. There are several useful rules relating to exponents. They are listed below together with an illustration of why they work.

| Rules | Illustrations |
|---|---|
| 1) $a^m \cdot a^n = a^{m+n}$ | 1) $x^2 \cdot x^3 = (xx)(xxx) = x^5 = x^{2+3}$ |
| 2) $\left(a^m\right)^n = a^{m \cdot n}$ | 2) $\left(y^4\right)^2 = \left(y^4\right)\left(y^4\right) = y^8 = y^{4 \cdot 2}$ |
| 3) $(ab)^m = a^m b^m$ | 3) $(xy)^3 = (xy)(xy)(xy) = x^3 y^3$ |
| 4) $a^{-m} = \frac{1}{a^m}$ (Definition) | 4) $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$ |

<u>Remark</u>: Don't forget to remove the negative sign after you take the reciprocal!

| Rules | Illustrations |
|---|---|
| 5) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ | 5) $\left(\frac{x}{y}\right)^4 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^4}{y^4}$ |
| 6) $\frac{a^m}{a^n} = a^{m-n}$ | 6) $\frac{x^3}{x^1} = \frac{xxx}{x} = \frac{xx}{1} = x^2 = x^{3-1}$ |
| | $\frac{y^2}{y^4} = \frac{yy}{yyyy} = \frac{1}{yy} = \frac{1}{y^2} = y^{-2} = y^{2-4}$ |
| 7) $a^0 = 1$ (Definition) | 7) $\frac{x^2}{x^2} = x^{2-2} = x^0$ and $\frac{x^2}{x^2} = 1$ |
| | $\Rightarrow x^0 = 1$ |
| 8) $a^{\frac{1}{2}} = \sqrt{a}$ | 8) $16^{\frac{1}{2}} = \left(4^2\right)^{\frac{1}{2}} = 4^1 = \sqrt{16}$ |

The following activity will help you practice using the various rules for exponents.

**Activity A3.1**

Evaluate or simplify the following expressions using the rules above. For each expressions, indicate which rules you used. Your answers should not contain any parentheses or negative exponents, and each base should show up only once in each product or fraction.

| | | |
|---|---|---|
| a) $2^2 \cdot 2^3 =$ | $10^7 \cdot 10^{-7} =$ | $3^{-2} \cdot 3^{-1} =$ |
| b) $\left(5^2\right)^6 =$ | $\left(3^{-2}\right)^2 =$ | $\left(10^2\right)^4 =$ |
| c) $\left(\frac{1}{2}\right)^4 =$ | $\left(\frac{4}{5}\right)^{-2} =$ | $\left(\frac{8}{5}\right)^2 =$ |
| d) $100^{\frac{1}{2}} =$ | $\left(\frac{1}{4}\right)^{\frac{1}{2}} =$ | $\left(\frac{64}{49}\right)^{\frac{1}{2}} =$ |
| e) $(3x)^{-3} =$ | $(xy)^4 =$ | $\left(x^2 y\right)^2 =$ |
| f) $\frac{x^6}{x^6} =$ | $\frac{y^5}{y^2} =$ | $\frac{x^5 y^8}{y^4} =$ |
| g) $\frac{x^6 y^3}{x^2 y^4} =$ | $(xy)^2 x^{-4}$ | $\left(x^3 y^{-2}\right)\left(x^{-4} t^3\right)$ |

Our current notation for exponents was first used by Rene Descartes in *Discours de la Methode* (1637). For example, the equation

$$4x^3 - 6x^2 = 2x + 3$$

was written by Descartes as

$$4x^3 - 6xx \propto 2x + 3 .$$

This was a major improvement over the notations used by Italian mathematicians just 100 years earlier. The algebraist Rafael Bombelli (1526-1572) would have written the above equation as

4Cm. 6Q aeqtur 2R p. 3,

whereas Girolamo Cardano (1501 - 1575) would have used

4 cubus aequantur 6 quadratus & 2 res & 3.